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Technical Notes

Singular Spectrum Analysis Applied to Time-Series Measurements in a Self-Excited Tube Combustor

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Introduction

PRACTICAL propulsion systems (such as rocket engines and thrust augmenters) are often plagued by thermoacoustic instabilities (TAIs) arising from the coupling of chamber acoustics and combustion heat release. This coupling is typically represented as a feedback loop established when oscillations in acoustic pressure and heat release occur in phase with each other, that is, when the Rayleigh criterion is satisfied [1]. In general, thermoacoustic excitation relies on locating the heat source such that the oscillations in particle velocity u' lead those of acoustic pressure p' by $\sim$90 deg, producing a positive Rayleigh index Ra [2].

To sustain a TAI, the instability must be driven to overcome the per-cycle acoustic losses of the system. This condition assumes that sound radiation and boundary-layer losses make up the bulk of acoustic losses [3]. TAI growth is typically simplified to be exponential in nature [1], even though its amplitude cannot physically increase indefinitely. As the amplitude of a TAI grows, nonlinear effects involving acoustics and heat release become increasingly important [4]. A constant amplitude regime is then reached, known as the limit cycle.

Thermoacoustic instabilities almost always rely on excitation of resonant modes of the combustion chamber. Practical combustors have many complex modes, which are typically excited simultaneously and are extremely sensitive to both initial and boundary conditions [5]. An in-depth review of combustion instabilities in practical combustion systems can be found in an edited volume by Lieuwen and Yang [6]. It is necessary to identify a simple system for which various parameters governing the onset and sustenance of combustion instabilities can be isolated and studied independently. In this context, one such device which produces TAIs is a resonant tube [7]. The most common resonant tube is the Rijke tube, a pipe that is acoustically open on both ends. For the measurements reported in this study, we employed a Schmidt tube, a resonant tube with a closed–open acoustic configuration.

Although resonant-tube combustors provide ample opportunity for examination of fundamental thermoacoustic processes, the research community is still limited by the range of available diagnostic techniques. Detection systems capable of measuring heat release parameters with high spatial and temporal resolution are scarce. Radical species such as hydroxyl (OH) and methylidene (CH) have been extensively used as approximate markers of combustion heat release [8,9]. Laser-induced fluorescence (LIF) measurements with high spatial resolution have been applied to detect OH and CH in combustion systems [10]. However, most experimental investigations employing LIF measurements use laser systems with repetition rates several orders of magnitude lower than the fundamental frequencies of TAIs. Picosecond time-resolved laser-induced fluorescence (PITLIF) was developed to facilitate time-series measurements of OH concentrations in combustion systems [11]. This technique can facilitate a wide range of temporally and spatially resolved OH measurements at data acquisition rates necessary to resolve transient thermoacoustic processes.

In this study, we employ the PITLIF technique to measure OH time series and a dynamic pressure sensor to obtain pressure time series in a Schmidt tube combustor. The stationary (limit cycle) and nonstationary (transient) portions of simultaneous hydroxyl and pressure time series are analyzed by employing singular spectrum analysis (SSA). The noise-reduction capabilities of SSA are discussed from the point of view of instability mitigation in propulsion systems.

Singular Spectrum Analysis

Statistical discrimination of nonstationary processes is important in the study of practical propulsion systems [12]. Traditionally, signals embedded in a noisy series have been analyzed by statistical pattern-recognition techniques. The extension of classical pattern-recognition techniques to experimental time series has been a problem of great practical interest. In many practical problems, the time series are actually realizations of complex nonstationary processes. Various models of complex nonstationary processes have been proposed in the literature.

In this context, an important analytical procedure used for signal-to-noise enhancement, data compression, and pattern recognition for any time series is singular spectrum analysis [13]. Traditionally used for short, noisy time series, SSA helps to separate the time series into components that can be classified as trends, oscillations, and noise. An important feature of SSA is that the underlying oscillations can be phase- and/or amplitude-modulated; moreover, the trends can be nonlinear. In the current study, we use SSA to extract local features of both stationary and nonstationary time series with a strong focus on combustion instability.

The starting point of SSA is to embed a time series $[X(t){:}t=1,\ldots,N]$ into a vector space of dimension $M<N$. This approach essentially separates the time series into lagged copies through a set of overlapping M-point "windows." These lagged copies are then used to create a new series $X^*(t)$ of M-dimensional vectors so that

$$X^*(t)=(X(t),X(t+1),\ldots,X(t+M-1)) \tag{1}$$

The vectors in $X^*(t)$ are referenced by $t=1,\ldots,N'$, where $N'=N-M+1$. The selection of M depends on balancing the amount of information captured within each window (achieved by choosing a large value for M) with some level of statistical confidence in that information (which requires a large N/M ratio).

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Following segmentation of the original time series, the principal extended directions of the sequence of vectors $[X^*(t): t = 1, \ldots, N']$ are projected into phase space [14]. An $M \times M$ covariance matrix C_X can be created directly from the time series; its entries c_{ij} are given by

$$c_{ij} = \frac{1}{N - |i-j|} \sum_{i=1}^{N-|i-j|} X(t)X(t + |i-j|) \tag{2}$$

The M eigenvalues λ_k and their corresponding eigenvectors ρ_k can then be determined from

$$C_X \rho_k = \lambda_k \rho_k \tag{3}$$

The eigenvalue λ_k corresponds to the partial variance in the direction of its associated eigenvector. The eigenvalues are plotted in decreasing order of variance to obtain signal-to-noise separation. From this plot, we obtain two regions: an initial region that contains most of the signal and a second region that corresponds to the noise. The former typically has a steep slope, whereas the latter has a much lower amplitude and is often referred to as the floor [14]. If the time series contains underlying oscillations, the signal will contain eigenvalue pairs, that is, eigenvalues of similar magnitude, that correspond to a particular frequency.

The eigenvectors ρ_k are called empirical orthogonal functions (EOFs) and are not pure sine/cosine functions as in the usual Fourier analysis. Each EOF has a corresponding principal component (PC) A_k that arises from projecting the time series onto the eigenvector; the PCs are then given by

$$A_k = \sum_{j=1}^{M} X(t + j - 1)\rho_k(j) \tag{4}$$

Because the eigenvectors corresponding to the two principal components of a pair are similar, the phase difference between the two PCs is ± 90 deg. Such phase quadrature has previously been observed for eigenvectors corresponding to eigenvalue pairs in climatic time series [15].

A time-series reconstruction $R_\chi(t)$ can be created by combining a set of EOFs with their corresponding PCs:

$$R_\chi(t) = \frac{1}{M_t} \sum_{k \in \chi} \sum_{j=L_t}^{U_t} A_k(t - j + 1)\rho_k(j) \tag{5}$$

where χ is the set of EOFs underlying $R_\chi(t)$. The normalization factor M_t, the lower bound of summation L_t, and the upper bound of summation U_t all take on different values depending on the indexed location between the middle part of the time series and its end points. The series of length N making up $R_\chi(t)$, labeled the reconstructed component, has the important property of preserving the phase of the time series; therefore, $R_\chi(t)$ can be superimposed with $X(t)$ for comparison. Partial reconstruction is relatively simple for time series which contain periodicity and/or well-defined oscillations. In this case, the correct partial reconstruction of the signal, that is, the reconstruction over the correct set of EOFs, yields an optimal signal-to-noise enhancement with respect to white noise. To perform a complete reconstruction of the signal or of its oscillatory component(s), several other methods (either heuristic or based on Monte Carlo ideas) have been proposed for signal-to-noise separation or for reliable identification of oscillatory trends [14,16,17]. However, in this study, we employ the simple model proposed by Vautard et al. [16].

Results and Discussion

The experimental facility for simultaneous hydroxyl and pressure measurements comprises a Schmidt tube and the PITLIF laser system. The Schmidt tube assembly consists of a main tube with fused quartz windows placed along its length. The overall length of the main tube can be increased by adding extenders of identical cross section and was maintained at 151.6 cm for this study. A plenum chamber serves as the inlet for the fuel–air mixture, and a sintered metal plate at the base of the tube serves as the acoustically closed boundary. The flame holder is a cordierite ceramic honeycomb similar to that used by Nord [18] with an open area of 68%. The flat flame was placed at approximately the axial center of the tube.

The PITLIF setup uses a multimode Nd:YVO_4 laser to pump a mode-locked Ti:sapphire laser. The output of the Ti:sapphire laser consists of infrared (IR) pulses at a repetition rate of 80 MHz. The IR pulses are passed through a flexible harmonic generator which outputs a frequency-tripled UV beam. This beam is focused to create the probe volume by a series of optical elements. Fluorescence photons are detected by a photomultiplier tube and gated using a photon-counting system [11]. A detailed description of the laser system is provided by Zhang et al. [19]. Pressure fluctuations are monitored using a dynamic pressure sensor.

Simultaneous hydroxyl and pressure time series were acquired at a sampling rate of 2000 Hz. After the flame was lit, data were taken for 30 s so as to capture both the initial period before instability and the actual onset of that instability. For convenience, we have defined the onset as the time at which pressure fluctuations became audible. The amplitude of the onset showed significant growth, but eventually reached a limit cycle. Once the limit cycle was established, time series for both OH and pressure were sampled for 1 s. The OH time series was measured at an axial position of 1.5 mm above the surface of the honeycomb burner, whereas the pressure time series was acquired at the inlet of the tube. The amplitude of pressure fluctuations was relatively large, with sound pressure levels near 147 dB. In this range, the primary frequency of oscillations corresponds to the first excited mode of the tube ($\sim$188 Hz). An example of the OH time series at the limit cycle is shown in Fig. 1.

Applying SSA to this time series results in several pairs of eigenvalues (each corresponding to a particular frequency) that dominate over the rest; this correlates to the signal, as separated from the noise floor. The eigenvectors for these eigenvalues can then be plotted to verify that they are indeed in quadrature. The reconstruction is based solely on the EOFs and PCs associated with these initial eigenvalues and shows excellent agreement with the OH time series, as depicted in Fig. 1. This behavior is expected, as the time series itself appears to be periodic. It is important to note that the number of paired eigenvalues used in a reconstruction depends on the desired outcome. If a larger range of frequencies is preferred, a high number of pairs can be retained. However, if computational efficiency is of importance, only one or two pairs of eigenvalues can be used in the reconstruction.

It should be noted that the eigenvalues (and thus the EOFs and PCs) are weakly dependent on the selection of M. A general guideline for the selection of M is to choose a value between $N/5$ and $N/10$ [15]. For the current study, a value of $M = N/10$ was used, as the smaller value of M allows for a higher degree of statistical confidence while still remaining large enough to capture the frequencies of interest.

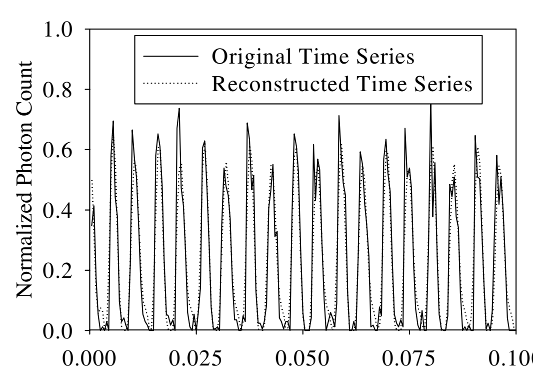

Fig. 1 Selected portion of OH time series at limit cycle and a reconstructed time series using SSA.

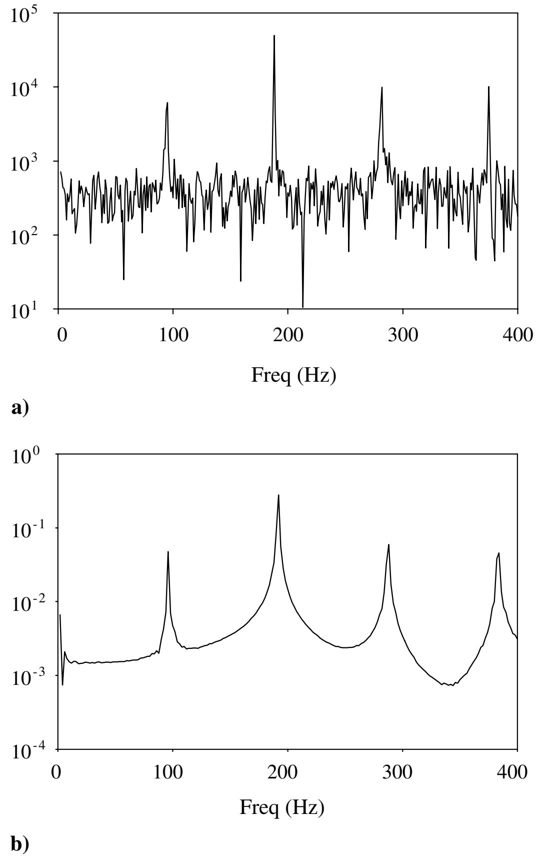

Fig. 2 Frequency spectrum for a) OH time series at limit cycle and b) its reconstruction.

To demonstrate the noise-reduction capability of SSA, a fast Fourier transform (FFT) was applied to both the original and reconstructed time series found in Fig. 1. The frequency spectrum of each was then plotted, as displayed in Fig. 2. Figure 2a shows the spectrum for the original OH time series, whereas Fig. 2b shows the spectrum for the reconstructed time series. Although peaks occur at the same excited frequencies (with ~188 Hz as the dominant frequency), the FFT of the reconstruction shows a large reduction in noise as compared to the original time series.

A similar analysis can be applied to the transient period both before and during the instability onset. Figure 3a shows a transient pressure time series. Figures 3b and 3c show portions of the original time series before and during onset of the instability, respectively. Although the amplitude of pressure fluctuations is quite small for the time series highlighted in Fig. 3b, analysis via SSA reveals the presence of 188-Hz oscillations. Figure 3d shows a time-series reconstruction using the first two principal components from the analysis of Fig. 3b. Figure 3e displays the time series reconstructed from Fig. 3c, also using just two principal components. The partially reconstructed time series shows good agreement with the measured time series. In particular, by employing only two principal components (corresponding to the first excited frequency at ~188 Hz), the SSA procedure is able to identify intermittent behavior before and during the instability onset. This highlights the ability of SSA to extract information from both a very noisy and nonstationary time series.

Conclusions

Thermoacoustic instabilities are of considerable interest owing to their prevalence in many practical combustion systems. A simple closed–open Schmidt tube combustor was employed to study the fundamental physics underlying these instabilities. Picosecond time-resolved laser-induced fluorescence was used along with a dynamic pressure sensor to obtain simultaneous time series for fluctuations in hydroxyl concentration and acoustic pressure. These time series were analyzed by using singular spectrum analysis, a data-adaptive method that can be applied to both stationary and nonstationary time series. Successful reconstruction and noise reduction were demonstrated, especially for extremely noisy and nonstationary time series. Future work will include the development of a hypothesis testing protocol to determine if SSA can predict the onset of instability.

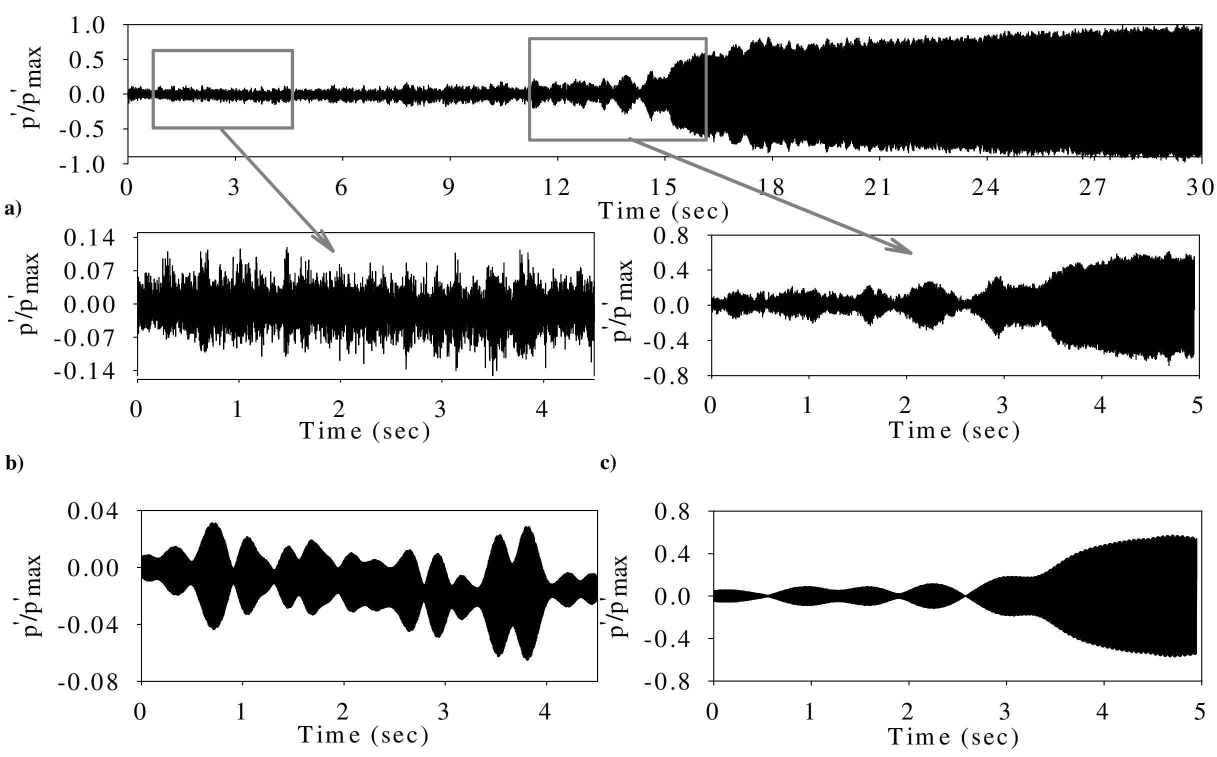

Fig. 3 SSA analysis of pressure fluctuations.

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